

An uncertainty analysis of the PVT gauging method applied to sub-critical cryogenic propellant tanks

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Abstract

The PVT (pressure, volume, temperature) method of liquid quantity gauging in low-gravity is based on gas law calculations assuming conservation of pressurant gas within the propellant tank and the pressurant supply bottle. There is interest in applying this method to cryogenic propellant tanks since the method requires minimal additional hardware or instrumentation. To use PVT with cryogenic fluids, a non-condensable pressurant gas (helium) is required. With cryogenics, there will be a significant amount of propellant vapor mixed with the pressurant gas in the tank ullage. This condition, along with the high sensitivity of propellant vapor pressure to temperature, makes the PVT method susceptible to substantially greater measurement uncertainty than is the case with less volatile propellants. A conventional uncertainty analysis is applied to example cases of liquid hydrogen and liquid oxygen tanks. It appears that the PVT method may be feasible for liquid oxygen. Acceptable accuracy will be more difficult to obtain with liquid hydrogen.

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1. Introduction and overview of measurement process

This paper explores the merit of the PVT method when applied to a cryogenic tank pressurized with helium gas during liquid expulsion. A conventional uncertainty analysis is applied to a thermodynamic model to obtain an estimate of the accuracy of the PVT method. Results are given for example cases of liquid hydrogen (LH₂) and liquid oxygen (LO₂) at tank expulsion pressures of 345 and 1030 kPa (50 and 150 psi).

PVT propellant gauging in low-g has been proven for storable propellants and is commonly used to predict end-of-life (propellant depletion) in earth-orbiting satellites [1,2]. The concept is based on conservation of helium in a propellant tank-helium bottle system (see Fig. 1). Pressure and temperature in the propellant tank and helium supply bottle are measured using the appropriate probes and sensors. The volumes of the tank and bottle are known. The measurement objective is to determine the quantity of liquid in the propellant

tank. PVT is a volumetric measurement which results in a liquid volume measurement, V_ℓ , when the ullage volume, V_u , is subtracted from a known total tank volume, V_t :

$$V_\ell = V_t - V_u \quad (1)$$

A classical thermodynamic analysis using Dalton's Law [3] is applied to the ullage mixture of propellant vapor and gaseous helium (GHe). Dalton's Law assumes each component exists separately at the volume and temperature of the mixture and that the sum of the component (partial) pressures equals the total pressure. If the ullage is well mixed and has uniform properties, then V_u is given by

$$V_u = m_{\text{He}} / \rho_{\text{He}} \quad (2)$$

where m_{He} and ρ_{He} are the mass and density of GHe in the ullage. The partial pressure of GHe in the ullage, P_{He} , is used to determine ρ_{He} . P_{He} is the difference of the total tank pressure P_t , and the vapor pressure of the propellant, P_v . For a well-mixed ullage, P_v is equal to the saturation pressure, P_{sat} , of the propellant at the ullage (or tank) temperature, T_u .

$$P_{\text{He}} = P_t - P_v = P_t - P_{\text{sat}}(T_u) \quad (3)$$

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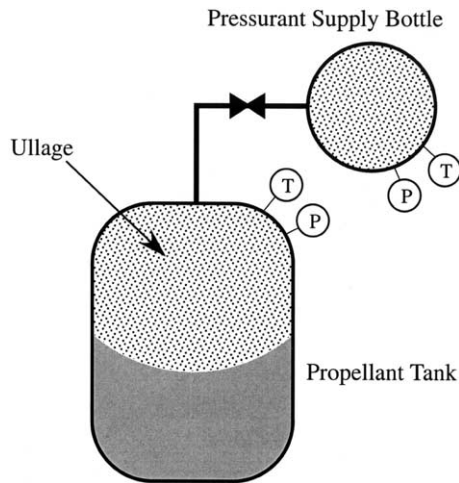


Fig. 1. Hardware configuration for PVT measurement.

Given an initial condition with no GHe in the ullage, m_{He} is equal to the mass transferred out of the bottle to the tank and is found by applying a mass balance to the GHe supply bottle.

$$m_{\text{He}} = (\rho_{b1} - \rho_{b2})V_b \quad (4)$$

where ρ_b is the GHe density in the bottle and V_b is the known bottle volume. The subscript 1 denotes an initial state and 2 denotes a later state after GHe has been transferred from the bottle to the tank. If temperature and pressure are measured in the bottle at states 1 and 2, the bottle densities, ρ_{b1} and ρ_{b2} can be determined and m_{He} calculated. Similarly for the tank, if temperature and pressure are measured, one can determine ρ_{He} for the ullage, then calculate V_u , and finally, V_ℓ .

The key requirements are: (1) bottle temperature is uniform and (2) ullage is well mixed, leading to uniform temperature and mixture composition. Requirement 1 might not be met if the bottle has hot and cold sides or immediately after bottle blow-down when the heat capacity of the bottle wall causes the wall temperature to lag behind the gas temperature. It should be possible to meet Requirement 1 using appropriate design and operational procedures. Requirement 2 will require active tank mixing that effectively eliminates thermal and species concentration gradients in the ullage—this could be a challenge at lower fill levels when the tank wall is not completely wetted by liquid propellant.

2. System definition and measurement values

Before applying an uncertainty analysis, it is necessary to define the system parameters and obtain representative values of all measured quantities required for the PVT measurement. The following assumptions are made for the system:

1. The propellant tank is initially 95% full, saturated at the initial pressure of 103 kPa (15 psi).
2. There is no GHe initially present in the tank.
3. Prior to liquid expulsion, the tank pressure is ramped to a higher level of 345 or 1030 kPa (50 or 150 psi) by injecting GHe from the supply bottle.
4. As liquid is expelled from the tank, the tank pressure is maintained at a constant value by injecting additional GHe.
5. The tank is modeled as an adiabatic control volume. The energy input is solely due to enthalpy inflow of the GHe. Heat leak and mixer power dissipation in a typical spacecraft tank are negligible in comparison, as is the heat capacity of the tank wall. Energy removal occurs solely due to enthalpy outflow during liquid expulsion.
6. The initial conditions in the GHe bottle are 20.7 MPa (3000 psi) and 278 K (500° R).
7. The minimum bottle pressure is 345 kPa (50 psi) above the tank expulsion pressure and the bottle is sized so that this pressure is reached when the tank fill level reaches 0 percent.
8. It is assumed that the thermal mass of the bottle wall cannot be neglected. The bottle is assumed to be spherical and constructed of titanium. The bottle wall thickness is calculated using the standard hoop stress equation for a thin-walled sphere using a factor of safety of 3.
9. The GHe in the bottle undergoes an isentropic blow-down as GHe is transferred to the tank. When outflow stops, the residual GHe and bottle wall then re-equilibrate at a new uniform temperature.

Four example cases were examined: (1) LO₂ tank expelled at 345 kPa (50 psi), (2) LH₂ tank expelled at 345 kPa (50 psi), (3) LO₂ tank expelled at 1030 kPa (150 psi), and (4) LH₂ tank expelled at 1030 kPa (150 psi).

A computer code was developed to generate the representative measurement values required as inputs for the PVT model. In each case, measurements are taken at the end of the pressurization ramp and 75%, 50%, 25%, and 0% fill. All thermodynamic properties are independent of tank size for a given bottle-to-tank volume ratio. (Even the bottle wall thermal mass scales directly with bottle volume. This is because the required wall thickness is proportional to bottle diameter.) Since the bottle volume scales directly with tank volume, a unit value of 1 m³ (35.3 ft³) was used for tank volume. It was not possible to determine the correct bottle size a priori, since the final GHe densities in the tank and bottle are temperature dependent and unknown. The code was exercised in a trial and error fashion until the specific bottle volume was found that gives the correct bottle pressure at the end of expulsion. All fluid prop-

erties for this analysis were obtained from a software package [4].

Representative values of the measured quantities are shown in Table 1 for Examples 1–4. The first data row for each example shows initial conditions and the optimized bottle size for a 1 m³ (35.3 ft³) propellant tank, while the second row lists conditions at the end of the pressurization ramp. The remaining rows contain expulsion data. At the end of the ramp, there can be a slight change in fill level due to compression or expansion of the liquid. Tank (ullage) temperature tends to increase as GHe is injected into H₂. The substantial tank temperature rise for the H₂ examples is due to the lower heat capacity of LH₂ on a unit volume basis and also the greater energy loss from the GHe due to a greater temperature drop as the GHe equilibrates with the propellant. Increasing the expulsion pressure increases the amount of required GHe and the bottle size. As will be seen later, the effect of increasing tank temperature during expulsion is undesirable for PVT because it leads to a reduced partial pressure of GHe. The tank temperature change is much less pronounced for O₂ due to its higher thermal mass and lesser amount of required GHe. Because the tank temperature for O₂ is about 67 K (120°R) higher than for comparable case with H₂, the

resulting GHe density is lower and less GHe is required for expulsion. This can be verified by comparing bottle volumes for Examples 1 and 2 or Examples 3 and 4. In all cases, the minimum bottle pressure occurring at the end of blow-down during expulsion from 25% to 0% fill was 345 ± 35 kPa (50 ± 5 psi) above the tank expulsion pressure. The final bottle pressures listed in the table exceed the tank expulsion pressure by more than 345 kPa (50 psi) due to temperature equilibration with the bottle wall after blow-down. The results tabulated in Table 1 are used as inputs to the uncertainty analysis.

3. Uncertainty analysis overview

We start by applying uncertainty methods to the equation for liquid propellant mass that gives the desired measurement in terms of directly and indirectly measured quantities. Eqs. (1) and (2) are combined to give the liquid volume:

$$V_\ell = V_t - m_{\text{He}}/\rho_{\text{He}} \quad (5)$$

Using conventional techniques [5] the uncertainty in V_ℓ can be expressed in terms of the measurement uncertainties of V_t , m_{He} , and ρ_{He} .

Table 1
Representative measured values for PVT gauging as predicted by adiabatic model

Tank (%) fill	Bottle-side			m_{He} (kg)	Tank-side		
	V_b (m ³)	P_b (MPa)	T_b (K)		V_t (m ³)	P_t (kPa)	T_u (K)
<i>Example 1, LO₂ @ 345 kPa</i>							
95.0	0.0463	20.7	278	—	1.00	103	90.4
95.0	0.0463	19.6	276	0.0637	1.00	345	90.5
75.0	0.0463	15.4	268	0.319	1.00	345	90.5
50.0	0.0463	10.7	260	0.639	1.00	345	90.4
25.0	0.0463	6.26	253	0.969	1.00	345	90.2
0.0	0.0463	1.64	248	1.36	1.00	345	89.3
<i>Example 2, LH₂ @ 345 kPa</i>							
95.0	0.0890	20.7	278	—	1.00	103	20.3
95.5	0.0890	18.6	274	0.229	1.00	345	20.9
75.0	0.0890	12.6	264	0.974	1.00	345	22.1
50.0	0.0890	8.21	256	1.58	1.00	345	22.7
25.0	0.0890	4.84	251	2.08	1.00	345	23.1
0.0	0.0890	1.49	247	2.63	1.00	345	23.2
<i>Example 3, LO₂ @ 1030 kPa</i>							
95.0	0.179	20.7	278	—	1.00	103	90.4
95.0	0.179	19.6	276	0.243	1.00	1030	90.7
75.0	0.179	15.5	269	1.20	1.00	1030	91.1
50.0	0.179	11.0	261	2.39	1.00	1030	91.4
25.0	0.179	6.79	253	3.59	1.00	1030	91.3
0.0	0.179	2.54	248	4.93	1.00	1030	89.9
<i>Example 4, LH₂ @ 1030 kPa</i>							
95.0	0.267	20.7	278	—	1.00	103	20.3
96.2	0.267	18.6	274	0.692	1.00	1030	22.3
75.0	0.267	12.3	264	3.06	1.00	1030	25.7
50.0	0.267	8.32	257	4.69	1.00	1030	27.1
25.0	0.267	5.29	251	6.04	1.00	1030	27.8
0.0	0.267	2.33	247	7.46	1.00	1030	28.1

$$U_{V_\ell}^2 = \left(\frac{\partial V_\ell}{\partial V_t} U_{V_t} \right)^2 + \left(\frac{\partial V_\ell}{\partial m_{\text{He}}} U_{m_{\text{He}}} \right)^2 + \left(\frac{\partial V_\ell}{\partial \rho_{\text{He}}} U_{\rho_{\text{He}}} \right)^2 \quad (6)$$

where U is the uncertainty of a measured or computed value. Eq. (5) is differentiated to obtain the various sensitivities that are then substituted into Eq. (6) to give:

$$U_{V_\ell}^2 = U_{V_t}^2 + \left(\frac{1}{\rho_{\text{He}}} U_{m_{\text{He}}} \right)^2 + \left(\frac{m_{\text{He}}}{(\rho_{\text{He}})^2} U_{\rho_{\text{He}}} \right)^2 \quad (7)$$

Next an uncertainty analysis is applied to the GHe bottle in order to estimate $U_{m_{\text{He}}}$. After that, a similar analysis is applied to the propellant tank to determine U_{V_t} and $U_{\rho_{\text{He}}}$. Eq. (7) can then be used to get an estimate of the PVT measurement uncertainty.

4. Bottle-side analysis

The amount of GHe transferred from the bottle to the tank was given by Eq. (4). Since GHe is single-phase, its density in the bottle is dependent on bottle temperature, T_b , and pressure, P_b . Therefore, the amount of transferred GHe mass and the uncertainty of this value are functions of the measured quantities— V_b , P_{b1} , T_{b1} , P_{b2} , T_{b2} . There is an additional source of uncertainty in the density value returned by the fluid properties subroutine—this uncertainty is estimated by the vendor to be 0.1–0.5% and is considered negligible in this work. It is necessary to use real gas properties; for example, GHe has a compressibility factor of 1.1 at the assumed initial state of 20.7 MPa, 278 K (3000 psi, 500°R).

The uncertainty, U , in m_{He} can be expressed in terms of the measurement uncertainties of V_b , P_b , and T_b .

$$U_{m_{\text{He}}}^2 = \left(\frac{\partial m_{\text{He}}}{\partial V_b} U_{V_b} \right)^2 + \left(\frac{\partial m_{\text{He}}}{\partial U_{P_{b1}}} U_{P_{b1}} \right)^2 + \left(\frac{\partial m_{\text{He}}}{\partial U_{T_{b1}}} U_{T_{b1}} \right)^2 + \left(\frac{\partial m_{\text{He}}}{\partial U_{P_{b2}}} U_{P_{b2}} \right)^2 + \left(\frac{\partial m_{\text{He}}}{\partial U_{T_{b2}}} U_{T_{b2}} \right)^2 \quad (8)$$

This is evaluated numerically. When using the numerical approximation method, care must be taken to be certain that the choice of perturbation values provides accurate derivative approximations. Each perturbation is reduced until the corresponding derivative converges.

The following assumptions were made for the measurement uncertainties:

- Bottle volume: $U_{V_b} = \pm 1\%$
- Pressure: $U_{P_b} = \pm 1\%$ of full scale
- Temperature: $U_{T_b} = \pm 1.4$ K (2.5°R)

The proper choice for measurement uncertainties requires good engineering judgment. In many cases, it will

be necessary to correct bottle volume for stretch, since volume changes of several percent are common in high pressure bottles. Pressure transducers are readily available with accuracies of one percent of full scale at room temperature. The assumed pressure measurement uncertainty may be optimistic under cold bottle conditions. Multiple cascaded pressure transducers could be used to get better accuracy at low bottle pressure. For now, only one transducer will be used and it is sized for the maximum bottle pressure. Unlike pressure, which is uniform throughout the bottle, it is possible or perhaps likely that there will be temperature gradients in the bottle. Therefore, the bottle temperature uncertainty will likely be greater than the accuracy of individual temperature sensors. It is assumed that PVT measurements will be made when sufficient time has elapsed after bottle blow-down so that the GHe and bottle wall have equilibrated. An actual bottle temperature measurement used for PVT should be an average of readings from distributed wall and internal sensors.

The assumed bottle measurement uncertainties are shown in the left side of Table 2. Uncertainties in the calculated transferred GHe mass ($U_{m_{\text{He}}}$) are included in the table. At the end of the ramp, the $U_{m_{\text{He}}}$ uncertainty is high in terms of percentage of transferred mass. There is a substantial reduction in percentage uncertainty as propellant tank fill level drops and more GHe is transferred to the tank. It is interesting to note that despite a high percentage uncertainty in the P_{b2} measurement at low bottle pressure (and low propellant tank fill level), $U_{m_{\text{He}}}$ has dropped to a few percent in each of the examples.

5. Tank-side analysis

Having obtained an estimate of $U_{m_{\text{He}}}$ from the bottle-side analysis, uncertainty estimates of the tank volume, U_{V_t} , and GHe density in the ullage, $U_{\rho_{\text{He}}}$, are needed to complete the estimate for liquid volume uncertainty given by Eq. (7). The tank volume uncertainty can be directly specified, but the uncertainty in helium density must be estimated from the uncertainties in the tank pressure and ullage temperature measurements. This was done numerically as:

$$U_{\rho_{\text{He}}}^2 \approx \left(\frac{\Delta \rho_{\text{He}}}{\Delta U_{P_t}} U_{P_t} \right)^2 + \left(\frac{\Delta \rho_{\text{He}}}{\Delta U_{T_u}} U_{T_u} \right)^2 \quad (9)$$

The following assumptions are made for the measurement uncertainties:

- Tank volume: $U_{V_t} = \pm 1\%$
- Pressure: $U_{P_t} = \pm 1\%$ of full scale
- Temperature: $U_{T_u} = \pm 0.56$ K (1°R) (includes allowance for ullage spatial variations)

Table 2
Uncertainty values of measured and computed quantities for PVT gauging

Tank (%) fill	Bottle-side						Tank-side				
	U_{V_b} (m ³)	$U_{P_{b1}}$ (kPa)	$U_{T_{b1}}$ (%)	$U_{P_{b2}}$ (kPa)	$U_{T_{b2}}$ (%)	$U_{m_{He}}$		U_{V_t} (m ³)	U_{P_t} (kPa)	U_{T_u} (%)	U_{V_t}
	±1%	±1% F.S.	±1.4 K	±1% F.S.	±1.4 K	±kg	±%	±1%	±1% F.S.	±0.55 K	±% F.S.
<i>Example 1, LO₂ @ 345 kPa</i>											
95.0	0.0005	207	0.5	207	0.50	0.0208	34	0.01	3.4	0.61	2.0
75.0	0.0005	207	0.5	207	0.52	0.0224	7.0	0.01	3.4	0.61	2.2
50.0	0.0005	207	0.5	207	0.53	0.0240	3.6	0.01	3.4	0.61	2.7
25.0	0.0005	207	0.5	207	0.55	0.0256	2.6	0.01	3.4	0.62	3.3
0.0	0.0005	207	0.5	207	0.56	0.0272	2.0	0.01	3.4	0.62	3.8
<i>Example 2, LH₂ @ 345 kPa</i>											
95.5	0.0009	207	0.5	207	0.51	0.0416	18	0.01	3.4	2.6	1.4
75.0	0.0009	207	0.5	207	0.53	0.0432	4.5	0.01	3.4	2.5	4.3
50.0	0.0009	207	0.5	207	0.54	0.0464	2.9	0.01	3.4	2.4	10
25.0	0.0009	207	0.5	207	0.55	0.0497	2.4	0.01	3.4	2.4	18
0.0	0.0009	207	0.5	207	0.56	0.0529	2.0	0.01	3.4	2.4	26
<i>Example 3, LO₂ @ 1030 kPa</i>											
95.0	0.0018	207	0.5	207	0.50	0.0833	35	0.01	10	0.61	2.0
75.0	0.0018	207	0.5	207	0.52	0.0849	7.1	0.01	10	0.61	2.1
50.0	0.0018	207	0.5	207	0.53	0.0897	3.8	0.01	10	0.61	2.3
25.0	0.0018	207	0.5	207	0.55	0.0961	2.7	0.01	10	0.61	2.6
0.0	0.0018	207	0.5	207	0.56	0.104	2.1	0.01	10	0.62	2.8
<i>Example 4, LH₂ @ 1030 kPa</i>											
96.2	0.0027	207	0.5	207	0.51	0.125	18	0.01	10	2.5	1.2
75.0	0.0027	207	0.5	207	0.53	0.131	4.3	0.01	10	2.2	2.6
50.0	0.0027	207	0.5	207	0.54	0.139	3.0	0.01	10	2	6.2
25.0	0.0027	207	0.5	207	0.55	0.146	2.4	0.01	10	2	11
0.0	0.0027	207	0.5	207	0.56	0.155	2.1	0.01	10	2	15

The specification for volume uncertainty must reflect that the volume of interest is fluid volume only and does not include volume displaced by internal hardware. One must also consider the significant volume shrinkage when a tank is cooled to cryogenic temperature. The tank pressure uncertainty specification should recognize that the measurement is obtained under cryogenic conditions. As will be seen shortly, the choice of ullage temperature uncertainty is the crux of the entire analysis. The key question is how spatially uniform the ullage temperature will be in a low-g cryogenic environment. The initial estimate of ±0.56 K (1°R) will certainly be optimistic if the ullage is not well mixed or if exposed to a warm tank wall. Reducing this uncertainty below the initial estimate will require very efficient fluid circulation and spray systems.

Representative tank-side values of measured quantities are provided in the right side of Table 2. The percentage uncertainty in ullage temperature is roughly 2–3% for LH₂ and less than 1% for LO₂. The resulting uncertainty in liquid volume (the end result of the PVT measurement) is shown in the rightmost column of Table 2. In all cases, the measurement uncertainty is least at high fill level and increases as the fill level drops. Based on the measurement technique and uncertainties as specified in the above discussion, it appears that a PVT measurement of less than 5% uncertainty can be

obtained at all fill levels for LO₂ tanks at an expulsion pressure of 345 kPa (50 psi). Increasing the expulsion pressure to 1030 kPa (150 psi) results in a further improvement for LO₂. Given the same set of assumptions, PVT does not appear to be an attractive gauging method for LH₂. In the low pressure LH₂ example, uncertainty is as high as 26%. In the high pressure case, maximum uncertainty is reduced to 15% which is still considered unfavorable for most applications.

6. Importance of various sources of uncertainty

In addition to estimating the overall uncertainty of the PVT measurement, the uncertainty analysis can determine the relative importance of each of the sources of uncertainty. This is useful for assessing the feasibility of making improvements to the PVT measurement. The uncertainty percentage contribution, UPC, from each measured quantity, X_i , to the overall uncertainty in the result, r , is defined as:

$$UPC_i = \frac{(\Delta r / \Delta X_i)^2 (U_{X_i})^2}{U_r^2} \times 100 \quad (10)$$

Table 3 lists the UPC for each measured quantity at various tank fill levels for each of the example cases. Results from the bottle-side are shown on the left side

Table 3

Uncertainty percentage contributions for measured and computed quantities in PVT gauging measurement

Tank (%) fill	Bottle-side					Tank-side			
	V_b	P_{b1}	T_{b1}	P_{b2}	T_{b2}	m_{He}	V_t	P_t	T_u
<i>Example 1, LO₂ @ 345 kPa</i>									
95.0	0	40	10	41	9	74	25	0	1
75.0	2	38	9	44	6	63	21	3	13
50.0	8	34	8	46	3	46	14	7	34
25.0	15	30	7	47	1	34	9	10	47
0.0	25	25	6	44	0	28	7	13	53
<i>Example 2, LH₂ @ 345 kPa</i>									
95.5	0	40	10	42	9	35	51	0	13
75.0	5	36	9	46	5	7	6	1	86
50.0	12	32	8	47	2	2	1	1	96
25.0	18	28	7	46	1	1	0	1	98
0.0	25	24	6	45	0	1	0	1	98
<i>Example 3, LO₂ @ 1030 kPa</i>									
95.0	0	40	10	41	9	75	25	0	0
75.0	2	38	9	44	7	73	23	2	3
50.0	7	35	8	46	4	67	19	6	8
25.0	14	30	7	47	1	60	15	10	14
0.0	26	26	6	45	0	55	12	15	18
<i>Example 4, LH₂ @ 1030 kPa</i>									
96.2	0	40	10	42	9	31	66	0	3
75.0	5	36	9	46	4	17	14	2	67
50.0	11	32	8	47	2	6	3	2	89
25.0	17	29	7	46	1	3	1	2	94
0.0	23	25	6	45	0	2	0	2	96

with the UPCs reflecting the impact on the calculated GHe mass transferred to the tank. The right side is a tabulation of the UPCs for the liquid volume measurement. It is important to recognize that improvements to the bottle side measurements have a minor or insignificant effect if the UPC for m_{He} is not a relatively large UPC on the tank-side. For Example 3, the UPC for m_{He} is dominant at all fill levels. Here it would clearly be worthwhile to examine the bottle-side to find the largest source of uncertainty, which for this example, is the measurement of P_{b2} . Improving the final bottle pressure measurement, perhaps by using a cascade of pressure transducers with different full scale ranges, would be worthwhile if a lower overall uncertainty is desired.

The dominant source of uncertainty for LH₂ at low fill levels (Examples 2 and 4) is the uncertainty of T_u . The T_u uncertainty is also large for the low pressure LO₂ case (example 1), but to a lesser extent. The overall measurement uncertainty at high fill levels is much lower and may be acceptable as is. In all cases, improvements at high fill level, if desired, require changes to the initial and final bottle pressure measurements, P_{b1} and P_{b2} .

7. GHe density uncertainty

The results indicate that PVT is more promising for LO₂ than for LH₂, and that the increased uncertainty

for the LH₂ measurement is mostly attributable to the uncertainty in ullage temperature. This difference between fluids can be explained by analyzing the uncertainty of the density of helium in the ullage, ρ_{He} . In the above examples, GHe has a compressibility factor nearly equal to one. Therefore one can reasonably model the GHe as an ideal gas. The following is an analytical expression for the uncertainty of ρ_{He} (the derivation is given in the Appendix A):

$$\left(\frac{U_{\rho_{He}}}{\rho_{He}}\right)^2 = \left(\frac{1}{T_u} + \frac{1}{P_{He}} \frac{dP_{sat}}{dT_u}\right)^2 U_{T_u}^2 + \left(\frac{1}{P_{He}}\right)^2 U_{P_t}^2 \quad (11)$$

The first term on the right contains the direct and indirect effects of temperature uncertainty in determining GHe density while the second term is the direct contribution from the tank pressure measurement. The indirect effect arises from the uncertainty of the vapor partial pressure as determined from temperature. (dP_{sat}/dT_u is the slope of the vapor saturation curve.) Increasing the GHe partial pressure will reduce GHe density uncertainty. (This is why Example 4 has lower PVT uncertainty than Example 2 and, to a lesser extent, why Example 3 shows an improvement over Example 1.) Increasing temperature will increase $U_{\rho_{He}}$ because dP_{sat}/dT_u increases and the indirect effect dominates the decrease from the direct effect. Fig. 2 shows a compar-

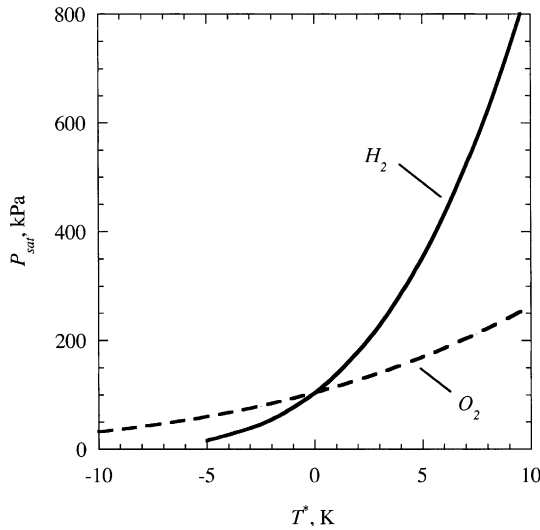


Fig. 2. Saturation pressure of H_2 and O_2 plotted vs normalized temperature, $T^* = T - T_{\text{sat}@103 \text{ kPa (15 psi)}}$.

ison of the saturation pressure curves for H_2 and O_2 . The temperature scale has been normalized by subtracting the saturation temperature at 103 kPa (15 psi) for each fluid [20.3 K (36.6°R) for H_2 and 90.4 K (162.7°R) for O_2]. The H_2 curve has a steeper slope; about 3–5 times greater in the region of interest for the example cases. As a result, the indirect effect of temperature uncertainty dominates Eq. (11) for H_2 , while the temperature and pressure terms are the same order of magnitude for O_2 in the region of interest. (This is why the improvement for Example 4 over Example 2 is substantial, while the improvement for Example 3 over Example 1 is modest.)

Some calculated results from Eq. (11) are shown in Fig. 3 where the uncertainty of ρ_{He} in percent is plotted

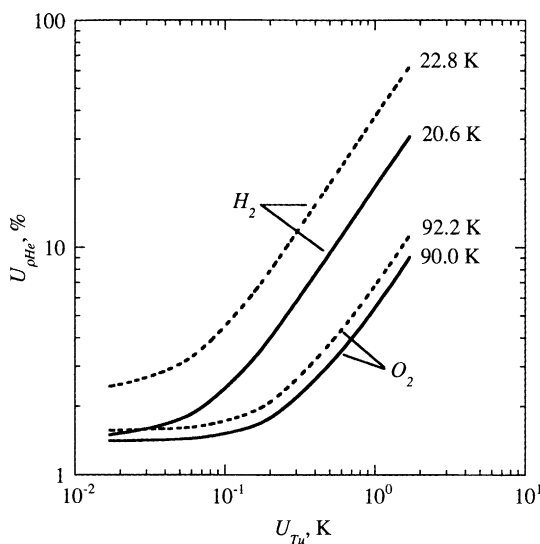


Fig. 3. Uncertainty of helium density in LH_2 and LO_2 tanks plotted vs uncertainty of ullage temperature measurement. [$U_{P_i} = \pm 3.5 \text{ kPa (0.5 psi)}$].

vs a range of uncertainty in T_u for a fixed tank pressure uncertainty of $\pm 3.5 \text{ kPa (0.5 psi)}$. The solid lines approximately represent the condition at the saturation temperature at 103 kPa (15 psi) for each fluid. The dashed lines represent conditions for a 2.2 K (4°R) increase in temperature, which based on Examples 1 and 2, is not likely for LO_2 , but quite possible with LH_2 . For a $\pm 0.56 \text{ K (1°R)}$ uncertainty in ullage temperature (the assumption used in the PVT analysis) there is about a 3–4% uncertainty in ρ_{He} for LO_2 , and 10–20% for the LH_2 case. The sensitivity to temperature is clearly greater for LH_2 as is evident from the spacing between the pairs of curves. As the curves show, reducing U_{T_u} will initially reduce $U_{\rho_{\text{He}}}$, but then the curves level off indicating no further benefit. The leveling off occurs when U_{P_i} becomes the dominant uncertainty. In the LH_2 case, it is not worthwhile to reduce U_{P_i} until U_{T_u} becomes something like $\pm 0.056 \text{ K (0.1°R)}$.

8. Effect of ullage temperature uncertainty on PVT uncertainty for a LH_2 tank

Since the ullage temperature uncertainty dominates the PVT results for LH_2 , it was of interest to explore this issue further. Example 2 [LH_2 expelled at 345 kPa (50 psi)] was re-analyzed with ullage temperature uncertainties, U_{T_u} , of ± 0.056 , ± 0.17 , ± 0.56 and $\pm 1.7 \text{ K (0.10°R, 0.30°R, 1.0°R and 3.0°R)}$ where $U_{T_u} = 0.56 \text{ K (1°R)}$ is the value assumed in the initial analysis. All other uncertainties remain the same as before. The results are plotted in Fig. 4 where the data symbols represent the assumed fill level locations corresponding to PVT measurements in the original example. The

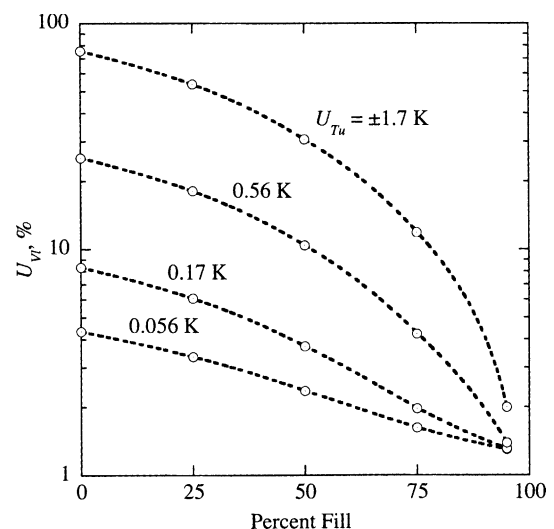


Fig. 4. Effect of ullage temperature uncertainty on overall PVT measurement uncertainty for LH_2 expelled at 345 kPa (50 psi) (All measured values and remaining measurement assumptions same as Example 2).

symbols are connected with dashed lines to aid interpretation of the results, but it should be understood that adding or removing measurement points as the tank is emptied will alter the tank temperature history somewhat and effect the results at following measurement points. The results show the high sensitivity of the overall PVT measurement uncertainty to U_{T_u} . If U_{T_u} is increased to ± 1.7 K (3°R), The PVT measurement uncertainty becomes much greater than 10% at all but the highest of fill levels and is clearly not acceptable for tank gauging. However, if U_{T_u} is decreased to ± 0.056 K (0.1°R), then uncertainty is less than 5% at all fill levels and appears to be an attractive gauging method. The key question is: “What is a reasonable value of ullage temperature uncertainty?” Clearly, obtaining values of less than 0.56 K (1°R) will require very effective propellant conditioning methods.

A cryogenic propellant tank can easily develop thermal stratification and ullage temperature gradients commonly occur. A tank mixer and spray system could be utilized to reduce ullage temperature uncertainty; the effectiveness of this active propellant conditioning system will determine whether PVT is a feasible gauging method or not.

9. Discussion

A study of the estimated measurement uncertainty of the PVT method when applied to cryogenic propellant tanks has been described. A model was developed to calculate representative temperatures and pressures in the propellant tank and helium bottle as the tank is pressurized and liquid is expelled. Four examples were studied— LH_2 and LO_2 each expelled at 345 kPa (50 psi) and at 1030 kPa (150 psi). PVT measurements were selected to occur at the end of tank pressure ramp-up and at fill levels of 75%, 50%, 25% and 0%. In practice, the actual temperatures and pressures are hardware-dependent and mission-specific but the models are thought to predict representative values for some typical conditions. The assumed measurement uncertainties were chosen to represent realistic values, although the ullage temperature uncertainty is subject to debate. The value chosen for U_{T_u} [± 0.56 K (1°R)] would require effective ullage conditioning and could be difficult to obtain in a ground-based test rig.

The analysis assumed no helium is initially present in the ullage and that initial bottle pressure and temperature are used in conjunction with current temperature and pressure measurements to calculate the liquid volume using the PVT method. The bottle temperature can be non-uniform immediately after blow-down; thus this would be a poor time to obtain reliable PVT measurements. The time required for the bottle contents to re-equilibrate with the bottle wall will depend on the bottle

material, thickness and other factors. In reality, conditions in the propellant tank will not be adiabatic as heat leak and mixer or pump power dissipation will vaporize propellant (much more so for LH_2 than for LO_2) and increase the vapor partial pressure at the expense of reduced GHe partial pressure unless this heat is removed by a thermodynamic vent or zero boiloff system. Reduction of GHe partial pressure has severe effects on PVT measurement uncertainty.

In some situations, use of higher tank pressure may be an option. If the ullage temperature is the same, but tank pressure is increased, then the vapor partial pressure will be increased and PVT uncertainty is reduced.

An uncertainty analysis is not an exact science, but can provide understanding when applied with good judgment and if all important sources of uncertainty are accounted for. There are at least two additional sources that may be of importance but have not been included in this paper. One is that mixtures of propellant vapor and GHe may not have ideal behavior; the other is that GHe is soluble to some extent in LH_2 and LO_2 . If the real gas properties of the vapor-GHe mixture are significantly non-ideal but can be characterized as functions of temperature and pressure, the PVT model can be improved to accommodate this. Similarly, relevant solubility data can be used to improve the PVT model to account for the GHe that goes into solution in the liquid propellant. A key issue here may be how the degree of solubility varies with time or with the nature of the fluid mixing process that is performed to achieve isothermal ullage conditions.

10. Conclusions

1. In all cases, the uncertainty is lowest at high fill levels and increases as fill level drops.
2. In all examples except the high pressure LO_2 case, overall PVT uncertainty is dominated by ullage temperature uncertainty when an ullage temperature uncertainty of ± 0.56 K (1°R) is assumed.
3. The high pressure LO_2 example has the lowest uncertainty and may be acceptable as is. If further improvements are desired, changes should be sought in the bottle pressure measurements.
4. All other things being equal, accurate PVT measurements are more feasible with LO_2 than for LH_2 .
5. With LH_2 , it is very important to minimize temperature rise in the ullage, as LH_2 has a significantly greater vapor pressure-to-temperature sensitivity than does LO_2 . An increase in temperature reduces GHe partial pressure and increases PVT uncertainty.
6. With an effective tank mixer and spray system, PVT accuracy of 5% of full scale or better appears feasible for LO_2 if the ullage temperature uniformity is on the order of 0.56 K (1°R) or less.

7. Raising the tank expulsion pressure reduces uncertainty for LO₂ tanks. Five percent accuracy could be met with some relaxation of the restraint on ullage temperature uniformity. Since the critical pressure for O₂ is high [5.07 MPa (736 psi)], it offers much “head-room” to reach favorable conditions for PVT.
8. To achieve 5% accuracy in LH₂, ullage temperature uniformity on the order of 0.056 K (0.1°R) or less is required when using warm GHe and tank pressures near 345 kPa (50 psi).
9. Raising the tank expulsion pressure reduces uncertainty for LH₂ tanks. Since the critical pressure of H₂ is 1.31 MPa (190 psi), the room for improvement for this fluid is more limited.
10. Further information on ullage temperature uniformity in tanks with mixing/spray systems is needed. Sufficient uniformity in low-g may be more readily obtained than in ground-based experiments due to reduced fluid buoyancy.
11. Further investigation into the non-ideal gas behavior of propellant vapor and GHe mixtures as well as the degree of GHe solubility in cryogenic liquid propellants should be performed prior to making final decisions about the use of the PVT method.

Appendix A. Derivation of the helium density uncertainty equation for the ullage

Since the GHe in the ullage can be modeled as an ideal gas ($Z_{\text{He}} \approx 1$), the density is given as:

$$\rho_{\text{He}} = \frac{P_{\text{He}}}{R_{\text{He}} T_{\text{u}}} = \frac{P_{\text{t}} - P_{\text{sat}}}{R_{\text{He}} T_{\text{u}}} \quad (\text{A.1})$$

Note that ρ_{He} is directly dependent on the measured quantities T_{u} and P_{t} . There is also an indirect dependence on T_{u} since the saturation pressure of the propellant, P_{sat} ,

is a function of temperature. The uncertainty in ρ_{He} is expressed as a function of T_{u} and P_{t} :

$$U_{\rho_{\text{He}}}^2 = \left(\frac{\partial \rho_{\text{He}}}{\partial T_{\text{u}}} U_{T_{\text{u}}} \right)^2 + \left(\frac{\partial \rho_{\text{He}}}{\partial P_{\text{t}}} U_{P_{\text{t}}} \right)^2 \quad (\text{A.2})$$

The partial derivatives are:

$$\begin{aligned} \frac{\partial \rho_{\text{He}}}{\partial T_{\text{u}}} &= \frac{1}{R_{\text{He}} T_{\text{u}}} \left(-\frac{P_{\text{t}}}{T_{\text{u}}} - \frac{dP_{\text{sat}}}{dT_{\text{u}}} + \frac{P_{\text{sat}}}{T_{\text{u}}} \right) \\ &= -\frac{\rho_{\text{He}}}{P_{\text{He}}} \left(\frac{P_{\text{He}}}{T_{\text{u}}} + \frac{dP_{\text{sat}}}{dT_{\text{u}}} \right) \end{aligned} \quad (\text{A.3})$$

and

$$\frac{\partial \rho_{\text{He}}}{\partial P_{\text{t}}} = \frac{1}{R_{\text{He}} T_{\text{u}}} = \frac{\rho_{\text{He}}}{P_{\text{He}}} \quad (\text{A.4})$$

Substituting (A.3) and (A.4) into (A.1) gives:

$$U_{\rho_{\text{He}}}^2 = \left[\frac{\rho_{\text{He}}}{P_{\text{He}}} \left(\frac{P_{\text{He}}}{T_{\text{u}}} + \frac{dP_{\text{sat}}}{dT_{\text{u}}} \right) \right]^2 U_{T_{\text{u}}}^2 + \left(\frac{\rho_{\text{He}}}{P_{\text{He}}} \right)^2 U_{P_{\text{t}}}^2 \quad (\text{A.5})$$

or:

$$\left(\frac{U_{\rho_{\text{He}}}}{\rho_{\text{He}}} \right)^2 = \left(\frac{1}{T_{\text{u}}} + \frac{1}{P_{\text{He}}} \frac{dP_{\text{sat}}}{dT_{\text{u}}} \right)^2 U_{T_{\text{u}}}^2 + \left(\frac{1}{P_{\text{He}}} \right)^2 U_{P_{\text{t}}}^2 \quad (\text{A.6})$$

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